ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 1 - 2016

Differentiation of Exponential and Trigonometric Functions

Resource Free

Name: SOLUTIONS **Teacher:** _____

Marks: 17

Time Allowed: 15 minutes

Instructions: You are NOT allowed any Calculators or notes.

You will be supplied with a formula sheet.

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1. Find $\frac{dy}{dx}$ *for*

a) $y = \frac{1}{2e^{3x}}$

$\frac{dy}{dx} = \frac{1}{2e^{3x}} \cdot -3$

$= \frac{-3}{2e^{3x}}$ ✓ ✓✓

b) $y = \cos(e^x)$

$\frac{dy}{dx} = -e^x \cdot \sin(e^x)$ ✓ ✓ ✓

c) $y = 3x^2e^{2x}$

$$\frac{dy}{dx} = 3x^2 \cdot 2e^{2x} + e^{2x} \cdot 6x \quad \checkmark \quad \checkmark$$
$$= 6x^2e^{2x} + 6xe^{2x}$$
$$= 6xe^{2x}(x+1) \quad \checkmark$$

d) $3\tan(1+e)^2$ $\quad \frac{dy}{dx} = 0 \quad \checkmark\checkmark$

[3,3,3,2 = 11 Marks]

2. Find the equation of the tangent to the curve defined by $h = (t^2 - 1)(t + 1)^8$:at the point (1,0).

$$\frac{dh}{dt} = (t^2-1)\,8(t+1)^7 + (t+1)^8(2t)$$
$$= 8(t^2-1)(t+1)^7 + 2t(t+1)^8 \quad \checkmark\checkmark$$
$$= 2(t+1)^7[4(t^2-1) + t(t+1)]$$
$$= 2(t+1)^7(4t^2 - 4 + t^2 + t)$$
$$= 2(t+1)^7(5t^2 + t - 4)$$
$$= 2(t+1)^7(t+1)(5t-4)$$
$$= 2(t+1)^8(5t-4)$$

When $t = 1$

$$\frac{dh}{dt} = 512 \quad \checkmark\checkmark$$

Now

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 512(x-1)$$
$$y = 512x - 512$$
$$\checkmark \quad \checkmark$$

[6 Marks]

ALL SAINTS' COLLEGE

MATHEMATICS DEPARTMENT

Year 12 Methods - Test Number 1 - 2016
Differentiation of Exponential and Trigonometric Functions
Resource Rich

Name: _____ **Teacher:** _____

Marks: 28

Time Allowed: 30 minutes

Instructions: You are allowed a ClassPad and 1 page of notes (both sides).

You will be supplied with a formula sheet.

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1) The population of a colony of numbats is being monitored by a group of scientists from Murdoch University. The population , P, after t years is modelled by the equation

$$P=4000e^{-0.01t}$$

a) What was the initial population of this colony of numbats?

4000 ✓

b) Find the exponential growth/decay of this colony?

decay of 1% per annum
✓ ✓

c) Find the population after 5 years?

When $t=5$, $P \approx 3804$
✓ ✓

d) After how many years will the population of numbats be half the size of the original population?

$2000 = 4000e^{-0.01t}$ ✓

$t \approx 69$ years ✓

[1,2,2,2 = 7 Marks]

2) An Olympic Ski Jumping slope has been designed so that it follows the curve:

$y = 3\cos(\frac{\pi x}{4}) + 8$ *for* $0 \le x \le 5$, *where x and y are both in metres.*

a) What is the take-off angle at the end of the jump (to the nearest degree) remembering that m = tan θ?

$\frac{dy}{dx} = -\frac{3\pi}{4}\sin(\frac{\pi x}{4})$ ✓✓

When $x = 5$

$\frac{dy}{dx} = -\frac{3\pi}{4}\sin\frac{5\pi}{4}$

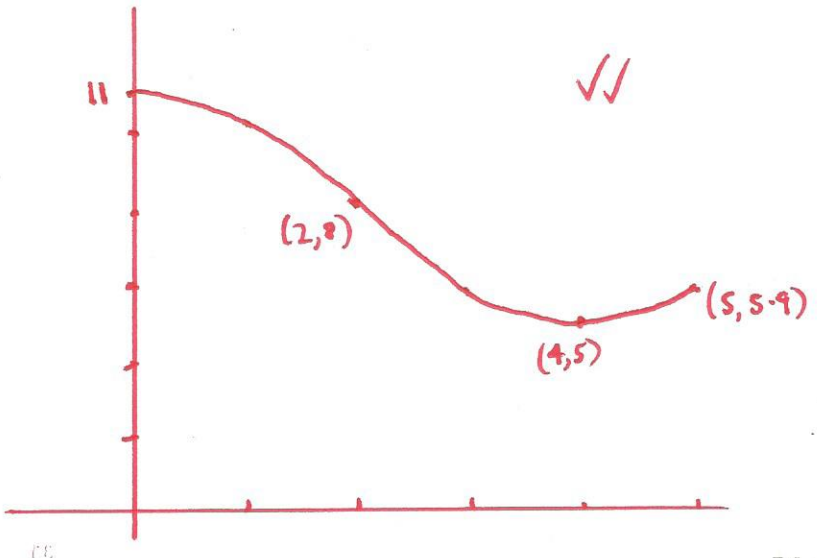
$= -\frac{3\pi}{4} \cdot \frac{-\sqrt{2}}{2} = \frac{3\sqrt{2}\pi}{8}$ ✓

$m = \tan\theta$

$\tan^{-1}(\frac{3\sqrt{2}\pi}{8}) \approx 59.0274°$ ✓

$= 59°$

b) Sketch the curve below:



✓✓

[4,2 = 6 Marks]

3) Western Australia is suffering from a decrease in average annual rainfall over time, t years, according to the formula $\frac{dR}{dt} = -.00975R$. The first average annual rainfall measured in WA was 880mm.

a) Find a formula for the average annual rainfall in this region.

$R = R_0 e^{-0.00975t}$ ✓✓

or $R = 880e^{-0.00975t}$

b) Find the average annual rainfall after:

i) 20 years

When $t = 20$

$R \approx 724.09$ mm ✓✓

ii) 100 years

When $t = 100$

$R \approx 331.93$ mm ✓✓

c) What is the rate at which the rainfall is decreasing after 100 years.

$R \approx 331.93$ mm

$\frac{dR}{dt} = -0.00975R$ ✓

$= -0.00975(331.93)$ ✓

$= -3.236$

⇒ Decreasing at around 3.236 mm per year after 100 years ✓

[2,2,2,3 = 9 marks]

4) Differentiate each of the following with respect to x:

a) $3x^2\tan(x)$

$3x^2(\tan x)^2 + 3x^2 + 6x\tan x$ ✓✓

$= 3x^2\sec^2 x + 6x\tan x$ or $= \frac{3x^2}{\cos^2 x} + 6x\tan x$

b) $[1+\cos(2x)]^4$

$-8(\cos(2x)+1)^3 . \sin 2x$ ✓✓

c) $\dfrac{e^{3x}}{(1-5x^2)}$

$\dfrac{-(15e^{3x}.x^2 - 10xe^{3x} - 3e^{3x})}{(5x^2-1)^2}$ ✓✓

[2,2,2 = 6 marks]

End of Test

Extra space for working out